Driver and Terminus Phase Relationships and the Impact on the System SPL Response

By Martin J King
MJKing57@aol.com

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Introduction

The calculated SPL system response for an unstuffed TL has many peaks and nulls, all the freeware software confirms this by producing very similar results. The plots are quite extreme when considering the desire for a flat SPL response across the low to mid frequency range. The interpretations and explanations found on the Internet for the peaks and nulls typically contain significant amounts of hand waving and incorrect conjecture.

To understand the peaks and nulls in the SPL plots, an understanding of the air motions in the TL and the volume velocity relationship between the driver end and the terminus end is required. The more you understand about the air motions in a TL, the easier it is to make intelligent design trade-offs. The intent of this presentation is to provide an explanation of the causes of the ragged SPL response using plots, minimal math, derived from one of my simplest MathCad models.

Transmission Line Speaker Model

To simplify the analysis and eliminate all other contributing variables to a TL's acoustic response, the following assumptions are made.

- The TL geometry has a constant cross-sectional area for the full length, no folds or bends are modeled.
- The driver is placed at the closed end of the TL to maximize the excitation of all quarter-wavelength standing waves.
- The boundary condition at the open end is set to the acoustic impedance for a piston in an infinite baffle, a common assumption. This boundary condition provides a mass loading at low frequencies transitioning to resistive damping as frequency increases.
- The model results are from one of my MathCad worksheets, similar results could be obtained from any of the other freeware TL simulation programs.

One inherent assumption in almost all freeware simulation programs is that the driver and terminus are coincident and radiate from an infinite baffle. Don't ask how this is physically possible, just recognize it as an assumption that will impact the following explanation.

MathCad Worksheet Inputs

Bass Driver Thiele / Small Parameters : Satori WO24P-4 Woofer

$f_d := 32.81 \cdot Hz$	$V_{ad} := 48.31 \cdot liter$
$R_e := 3.3 \cdot \Omega$	$Q_{ed} := 0.545$
$L_{vc} := 0.372 \cdot mH$	$Q_{md} := 7.637$
$R_p := 0.74 \cdot \Omega$	
$L_p := 0.595 \cdot mH$	
$Bl := 7.524 \cdot \frac{newton}{amp}$	$Q_{td} := \left(\frac{1}{Q_{ed}} + \frac{1}{Q_{md}}\right)^{-1}$
$S_d := 255 \cdot cm^2$	$Q_{td} = 0.509$

Enclosure Geometry Definition (Straight TL) $\frac{cycle}{4} \cdot \frac{c}{f_d} = 103.195\,in$

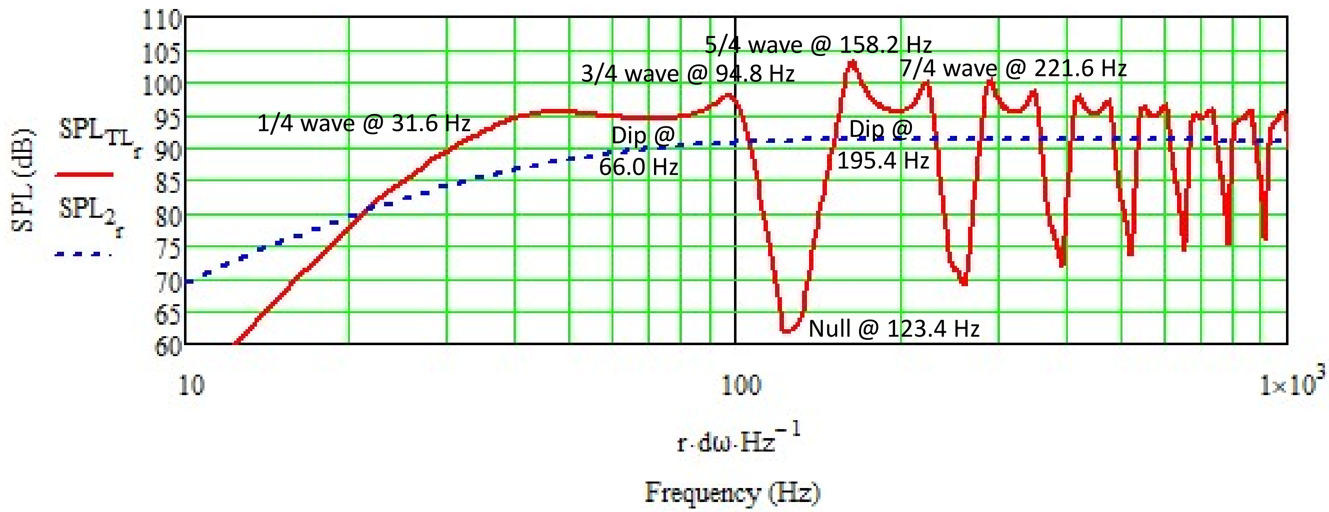
$L := 103 \cdot in$	(Length)	Driver Distance
$\xi := 0.001$	(Driver Position Ratio : $0.001 < \xi < 0.999$)	$\xi \cdot L = 0.10\,in$
$S_0 := 1.5 \cdot S_d$	(Area of the Closed End)	TL Volume
$S_L := 1 \cdot S_0$	(Area of the Open End)	$S_0 \cdot L = 100.070\,liter$
$Density := 0.05 \cdot lb \cdot ft^{-3}$	(Stuffing density : $0\ lb/ft^3 < D < 1\ lb/ft^3$)	
$Power := 1 \cdot watt$	(Input Power) Applied Voltage Reference --->	$R_{ref} := 8 \cdot \Omega$

A Satori WO24P-4 woofer is modeled in a 103-inch-long straight TL with an internal volume of 100 liters. The TL's physical length is equal to a quarter-wavelength at the driver's resonant frequency. The final tuning frequency will be slightly lower due to the terminus acoustic boundary condition.

There is no driver offset and only a very light distributed fiber damping (0.05 lb/ft^3) to control and limit the peaks and nulls in the SPL response.

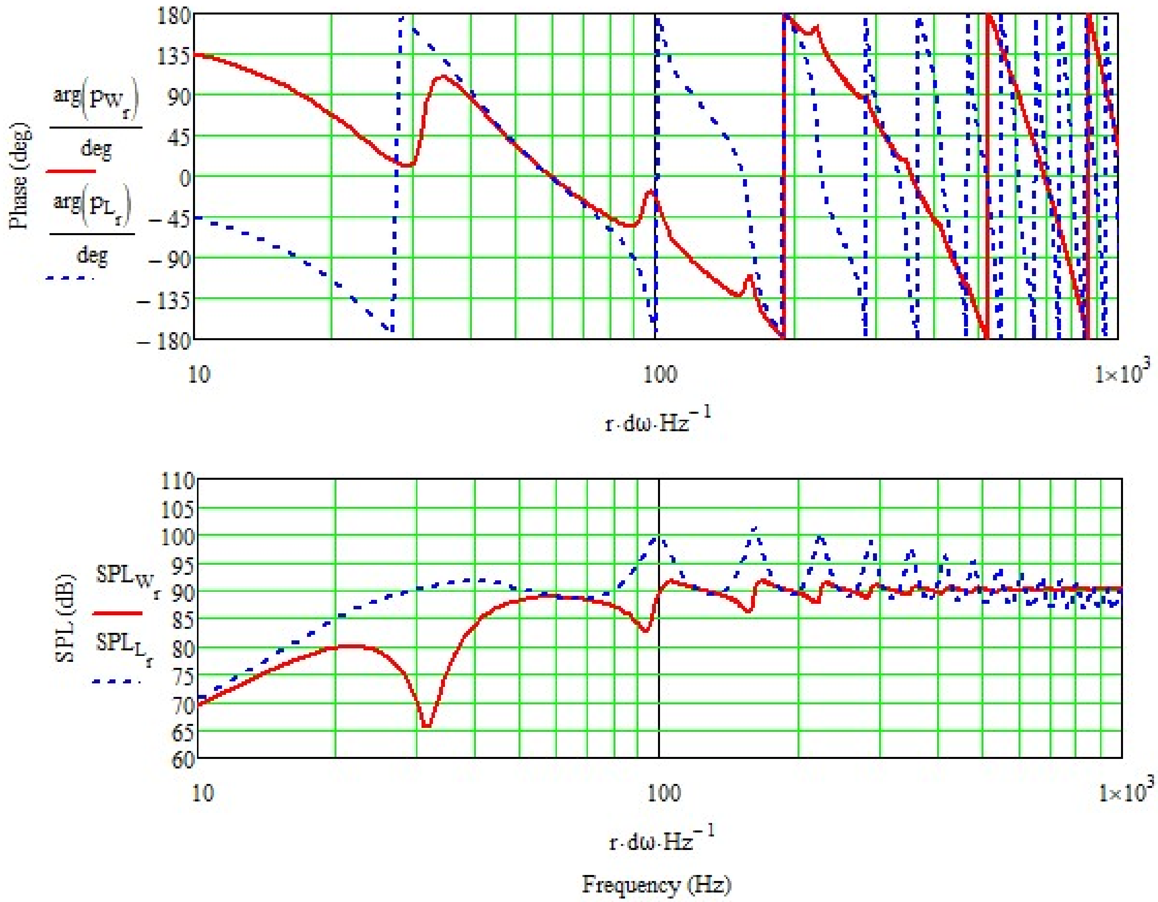
Calculated SPL Responses

System Response



The driver's infinite baffle response is shown as the dashed blue curve and the TL's response as the solid red curve. At the tuning frequency, the TL's bass response has been reinforced compared to the infinite baffle response. Above about 100 Hz, a pattern of quarter-wave peaks is seen with alternating deep nulls and shallow dips between them. Also note that the shallow dip still produces more output than the infinite baffle response. Recognize that the above plot is the sum of the coincident driver and terminus SPL outputs for an infinite baffle at a 1 m distance.

Driver and Terminus Responses



The driver's SPL and phase responses are shown as the solid red curves while the terminus SPL and phase responses are shown as the dashed blue curves. The combination of the driver and terminus responses produces the plot on the previous slide.

The magnitudes and phases of each of these responses are tabulated on the following slide for the specific frequencies identified on the previous slide. This table helps explain the peaks, dips, and nulls seen in the system SPL curve.

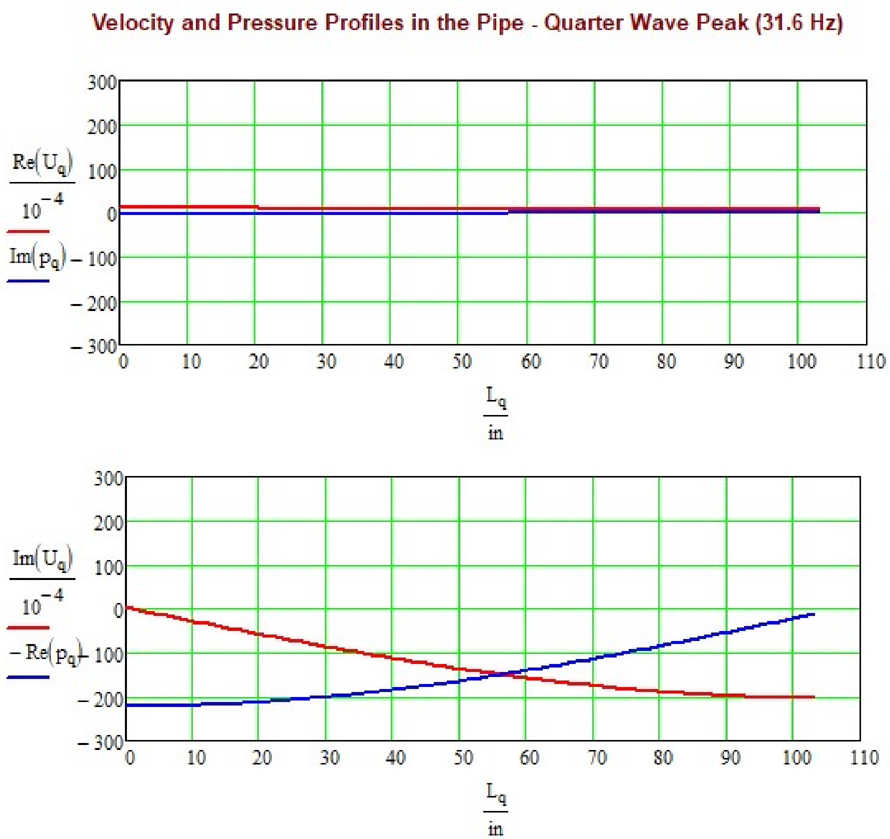
		Driver		Terminus		Calculated	Theoretical
Label	Frequency	Magnitude	Phase	Magnitude	Phase	Delta-Phase	Delta-Phase
1/4 Wave	31.6	65.1	65.0	90.6	150.0	-85.0	-90
Dip	66.0	88.4	-18.2	88.4	-22.0	3.8	0
3/4 Wave	94.8	82.9	-27.2	97.6	-120.9	93.7	90
Null	123.4	90.0	-99.5	89.7	81.8	181.3	180
5/4 Wave	158.2	86.9	-112.5	100.1	-24.8	-87.7	-90
Dip	195.4	89.6	173.3	89.3	-169.8	3.5	0
7/4 Wave	221.6	88.5	165.1	99.7	75.5	89.6	90
Units	[Hz]	[dB]	[deg]	[dB]	[deg]	[deg]	[deg]

There is a pattern in the magnitudes and phase relationships between the driver and terminus SPL output.

- At the 1/4 wavelength frequency, and the odd harmonics, the driver magnitude is significantly attenuated and the terminus magnitude peaks, they are +/- 90 degrees out of phase.
- At the 1/2 wavelength frequency, and the even harmonics, the driver and terminus outputs are almost equal, they are in or out of phase leading to reinforcement at the dips and almost complete cancellation at the nulls. The TL's SPL at the dips is almost 6 dB more than the infinite baffle SPL response which is consistent with the terminus's output being in phase and reinforcing the driver's output.

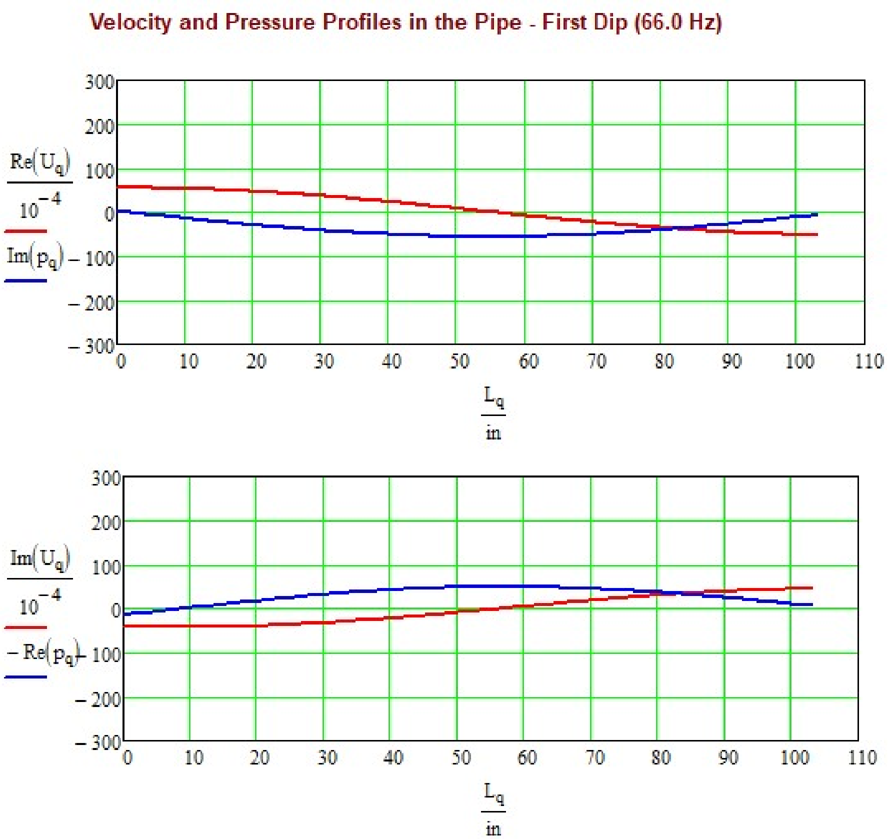
While this is still kind of a black box explanation with the driver response as the input and the terminus response as the output, plotting the pressure and volume velocity profiles along the TL's length should help in gaining a better picture of the physics. Understanding the air motions inside the TL enclosure will lead to more intelligent trade-offs and better TL designs.

Plotted Velocity and Pressure Profiles along the TL's Length



The volume velocity (red curves) and pressure (blue curves) have a magnitude and phase. The magnitudes and phases are resolved and plotted as real (Re) and imaginary (Im) curves along the TL's 103-inch length.

In the lower curve a quarter-wave shape is seen in both the volume velocity and pressure profiles. At the driver end (x = 0 inches) the driver volume velocity is approaching zero and is a maximum at the open end (x = 103 inches). The pressure response is doing the opposite, maximum at the closed end and essentially zero at the open end. This is consistent with the SPL outputs plotted on slide 7 at a frequency of 32.6 Hz.

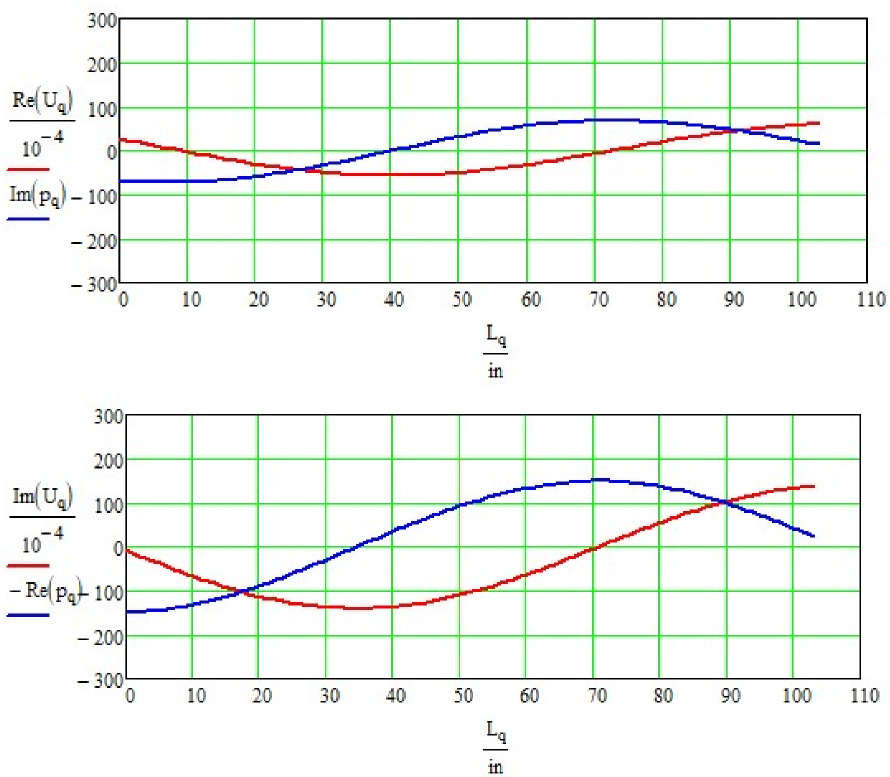


Looking at both the volume velocity and pressure curves you can see a half wavelength shape at 66.0 Hz. The pressure curve is a half sine wave that approaches zero at each end of the TL. The volume velocity curve is a half cosine wave with maximums at each end of the TL.

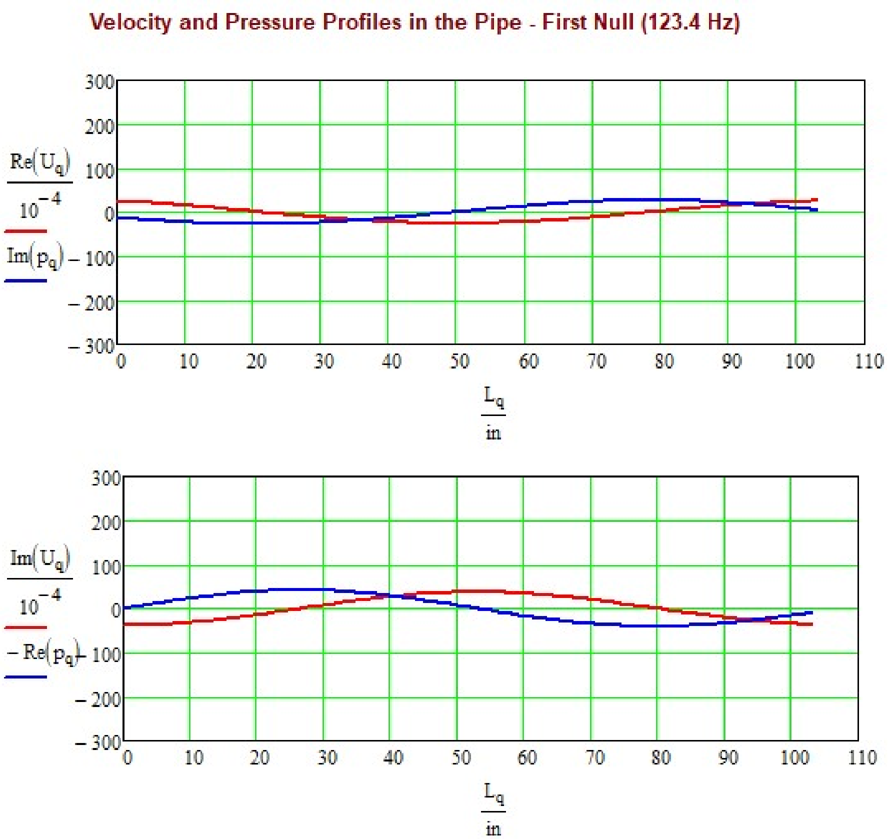
The volume velocity curve has the same magnitude at each end of the TL. The sound wave is constrained to travel along a constant area, so it does not spread and decrease in amplitude with distance like a simple source radiating into free space.

Inside the TL, the volume velocity at the back of the driver's cone is 180-degrees out of phase with the terminus end. The volume velocity at the front of the driver's cone is 180-degrees out of phase with volume velocity at the back of the cone. This leads to the driver and the terminus output into the room being in phase and adding constructively producing about 6 dB of extra output compared to the IB output.

Velocity and Pressure Profiles in the Pipe - Three Quarter Wave Peak (94.8 Hz)



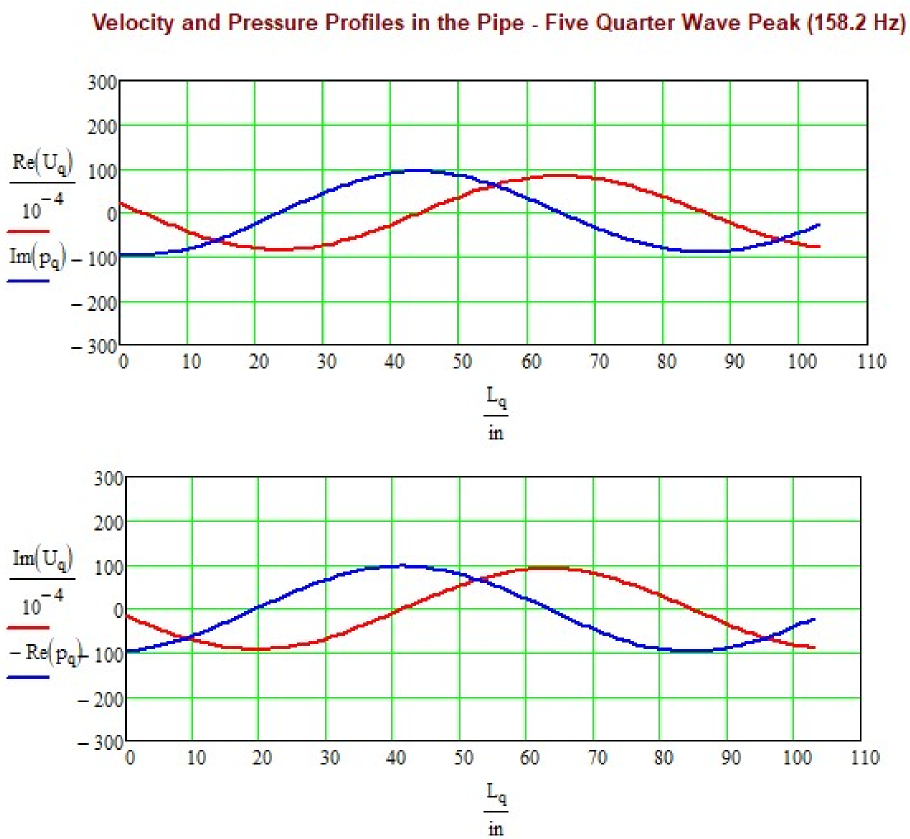
In the both sets of curves a three quarter-wave shape is seen for the volume velocity and pressure profiles. At the driver end (x = 0 inches) the driver volume velocity is approaching zero and reaches a maximum at the terminus end (x = 103 inches). The pressure response is doing the opposite, maximum at the driver end and essentially zero at the terminus end. This is consistent with the SPL outputs plotted on slide 7 at the frequency of 94.8 Hz.



Looking at both the volume velocity and pressure curves you can see a full wavelength shape. The pressure curve is a complete sine wave that approaches zero at each end of the TL. The velocity curve is a complete cosine wave with a maximum at each end of the TL.

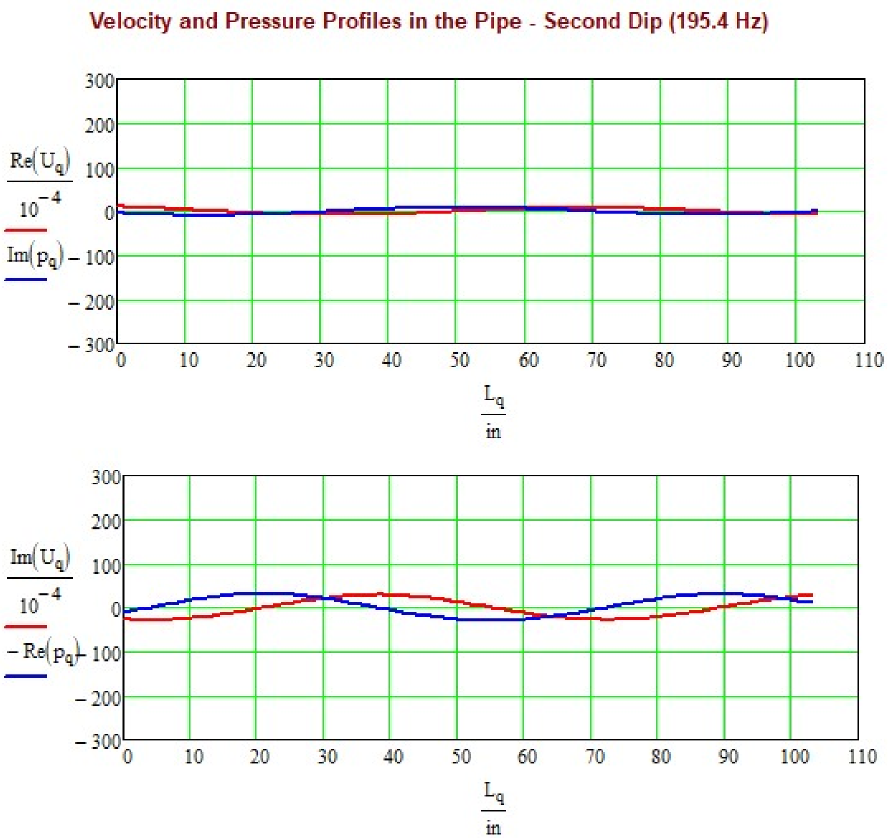
Again, the volume velocity curve has the same magnitude at each end of the TL. The sound wave is constrained to travel along a constant area, so it does not spread and decrease in amplitude with distance like a simple source radiating into free space.

Inside the TL, the volume velocity at the back of the driver's cone is in phase with the terminus end. The volume velocity at the front of the driver's cone is 180-degrees out of phase with the volume velocity at the back of the cone. This leads to the driver and the terminus output into the room being out of phase and adding destructively producing the deep null in slide 6 at 123.4 Hz.



In the both sets of curves a five quarter-wave shape is seen for the volume velocity and pressure profiles. At the driver end (x = 0 inches) the driver volume velocity is approaching zero and reaches a maximum at the open end (x = 103 inches). The pressure response is doing the opposite, maximum at the closed end and essentially zero at the open end. This is consistent with the SPL outputs plotted on slide 7 at the frequency of 158.2 Hz.

For all the quarter wavelength frequencies the speaker's output is produced primarily by the terminus end of the TL, the driver contributes very little output at these frequencies.

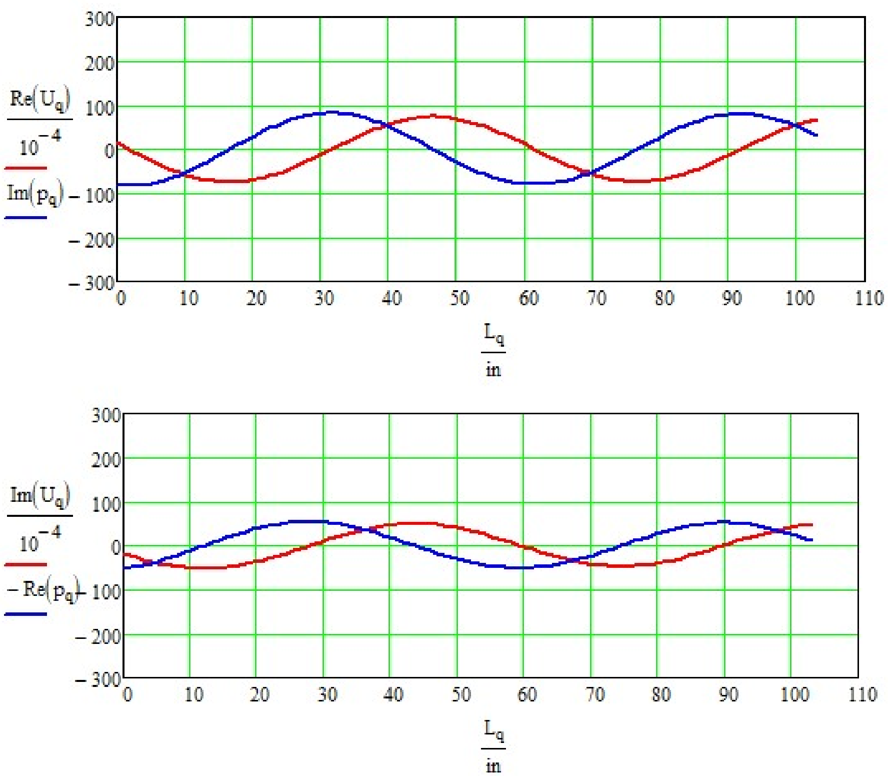


Looking at both the volume velocity and pressure curves you can see a three-halves wavelength shape. The pressure curve is one and a half sine waves that approaches zero at each end of the TL. The velocity curve is one and a half cosine waves with a maximum at each end of the TL.

Notice that the magnitudes of the half wave curves are getting less and less with increasing frequency compared to the quarter wavelength standing waves.

Inside the TL, the volume velocity at the back of the driver's cone is 180-degrees out of phase with the terminus end. The volume velocity at the front of the driver's cone is 180-degrees out of phase with the volume velocity at the back of the cone. This leads to the driver and the terminus output into the room being in phase and adding constructively producing about 6 dB of extra output compared to the IB output.

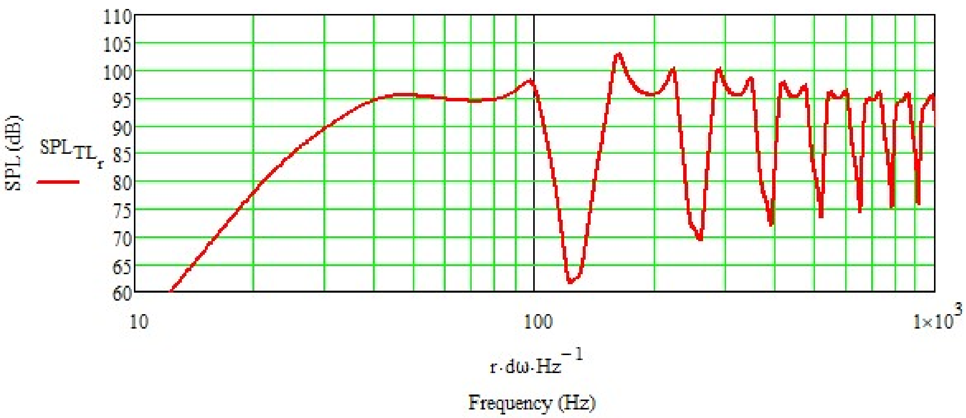
Velocity and Pressure Profiles in the Pipe - Seven Quarter Wave Peak (221.6 Hz)

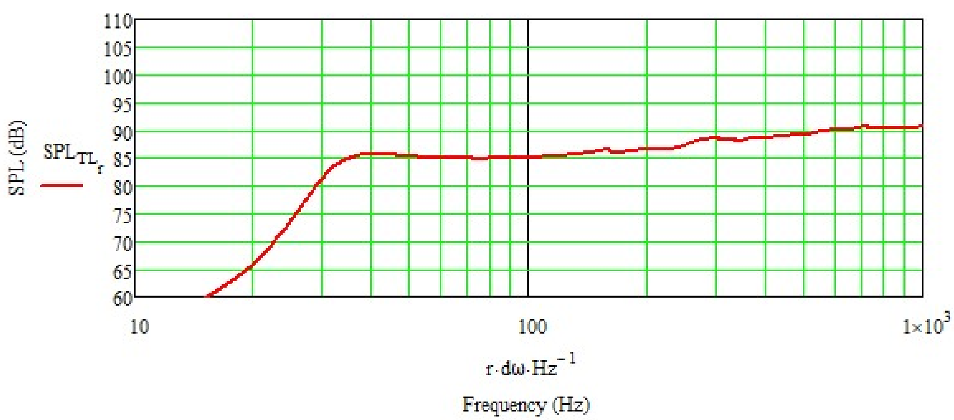


In the both sets of curves a seven quarter-wave shape is seen for the volume velocity and pressure profiles. At the driver end (x = 0 inches) the driver volume velocity is approaching zero and reaches a maximum at the open end (x = 103 inches). The pressure response is doing the opposite, maximum at the closed end and essentially zero at the open end. This is consistent with the SPL outputs plotted on slide 7 at the frequency of 221.6 Hz.

This pattern in the preceding plots repeats as frequency increases. The magnitudes of the local responses continue to decrease as the terminus acoustic boundary condition provides more damping at higher frequencies.

Simulating Reality





The upper plot is the SPL response for the straight end loaded empty TL described on slide 5. The results are ugly, no other way of describing them. From the previous slides, an explanation of the causes of the peaks, dips, and nulls has been provided. A lot of fiber stuffing could be added but the SPL response would still be a rolling affair with repeated humps and dips. This is a typical result of the available freeware TL design software. Thinking about it, how do you achieve a coincident driver and terminus, without a fold, on an infinite baffle in the real world? The freeware simulations are limited in accuracy.

The lower plot is also for an empty equivalent TL, but the geometry has been optimized. The driver is offset, the line has a 10:1 taper, the folds are modeled with the advance corner modeling method, and the external geometry of the enclosure is simulated (driver and terminus located at different positions on the front and back baffles). The baffle step between 200 and 600 Hz needs to be addressed by the crossover, it is not a function of a TL. Other than the baffle step, the SPL response is smooth and only a tiny bit of stuffing would be required to completely knock down the one or two small ripples. No significant amounts of fiber or foam damping, only geometric optimization is needed to remedy the ugliness in the upper plot.

Key Take Aways

- The freeware simulations of empty TL's and BLH's will produce peaks, dips, and nulls in the SPL response. The pattern of these will be the same in all quarter wave geometries and are determined by the sound's half wavelengths between the more dominant quarter wavelength standing waves. Hopefully, when you look at one of these plots presented on the Internet you now have a sense of what is going on in the enclosure.
- The end loaded TL SPL's results can be dramatically improved by offsetting the driver and tapering the area along the length, easy to model in all freeware programs. Assigning a distance between the driver and terminus outputs is also important but only available in Hornresp to the best of my knowledge.
- Adding the actual enclosure geometry to allow the baffle step to be estimated and using the advanced corner modeling method are two more improvements in the simulation accuracy. I do not believe these two features are available in any of the readily available free or commercial TL simulation programs.
- As a last resort fiber stuffing or foam can be used to brute force the ragged response to become somewhat smoother. Unfortunately, this also throws away some low-end extension. No free lunch for the TL designer.

Attachment

Early Explanation from 2018

Assume we have a driver with an fs of 50 Hz and it is installed at the closed end of a straight TL tuned to 50 Hz, the simplest form of TL design. The TL will produce standing waves at the 1/4, 3/4, 5/4, … frequencies of 50 Hz, 150 Hz, 250 Hz, … as expected. Neglect the acoustic impedance at the open end or terminus and assume the TL is completely empty so there is no damping of any kind. At a standing wave resonance, the back pressure on the driver cone will attenuate the motion, almost stopping the driver like in a BR design, and almost all of the SPL output will be from the terminus, like the port in a BR enclosure. The SPL and phase of the outputs will behave as follows.

Well below 50 Hz – as the driver moves into the TL it displaces a volume of air and an equivalent volume of air is pushed out of the terminus. The driver and the terminus are 180 deg out of phase and the SPL almost cancels producing a 24 dB/octave roll-off below 50 Hz.

At 50 Hz – the 1/4 fundamental standing wave is excited, the driver motion is significantly attenuated, and most of the SPL output comes from the terminus. The driver and the terminus are 90 degrees out of phase.

At 100 Hz – the driver and terminus are now in phase. There is no standing wave and the SPL from the driver and terminus are equal. This is because sound radiated from the back of the cone is the same as from the front of the cone but 180 degrees out of phase, the sound traveling down the TL is constrained so it does not decrease with distance, and the distance it travels is equal to a half of a wavelength (another 180 degrees). Theoretically, the system SPL will be 6 dB greater than the driver's SPL in an infinite baffle.

At 150 Hz – the 3/4 standing wave is excited, the driver motion is again significantly attenuated, and most of the SPL output comes from the terminus. The driver and the terminus are 270 degrees (-90 degrees) out of phase.

At 200 Hz – the driver and terminus are now out of phase. There is no standing wave and the SPL from the driver and terminus are equal. This is because sound radiated from the back of the cone is the same as from the front of the cone but 180 degrees out of phase, the sound traveling down the TL is constrained so it does not decrease with distance, and the distance it travels is equal to a full wavelength (another 360 degrees). The driver and the terminus are 180 deg out of phase and the SPL almost cancels producing a deep null, maybe this is what the measurements are showing.

The phase and SPL pattern repeat as frequency increases and moves through the higher quarter wave frequencies. Below is a list of key frequencies and the phase differences between the driver and terminus.

10 Hz – 180 deg SPL --> 0 dB

50 Hz – 90 deg fundamental 1/4 wave and SPL mostly from terminus

100 Hz – 0 deg SPL + 6 dB

150 Hz – 270 deg (-90 deg) 3/4 wave and SPL mostly from terminus

200 Hz – 180 deg SPL --> 0 dB

250 Hz – 90 deg 5/4 wave and SPL mostly from terminus

300 Hz – 0 deg SPL + 6 dB

350 Hz – 270 deg (-90 deg) 7/4 wave and SPL mostly from terminus

400 Hz – 180 deg SPL --> 0 dB

The pattern repeats in steps of 50 Hz. I hope that is clear