Modeling Method for Two Drivers in a Transmission Line

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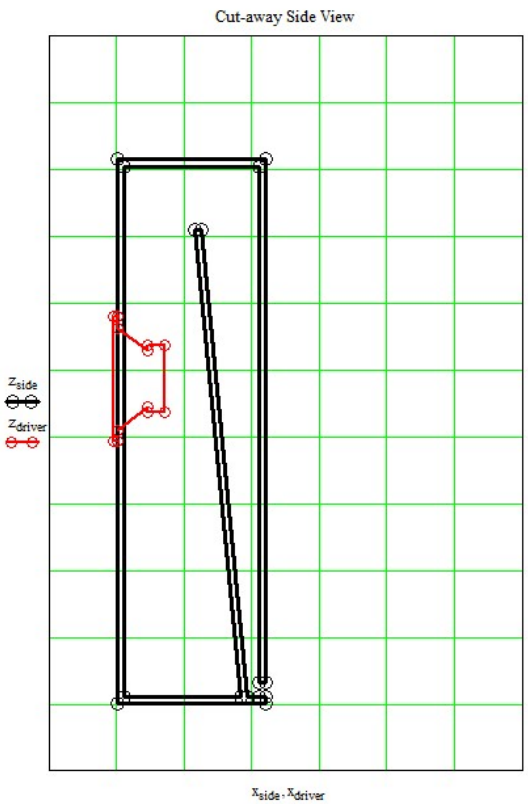
Introduction

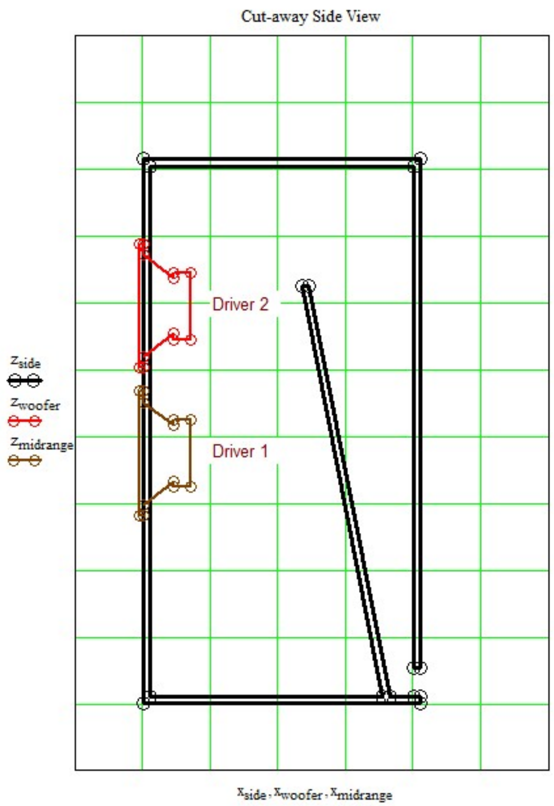
The theory for modeling a single driver in a sealed, bass reflex, transmission line, or horn enclosure has been derived and documented. Equivalent acoustic and electrical circuits can be found in textbooks (Beranek's acoustics texts are excellent references), AES papers (the Thiele and Small papers in particular), and independent websites around the Internet. Based on these equivalent circuit models, accurate simulation of a single driver in an enclosure can be performed using freeware and/or commercial loudspeaker design programs.

Modeling two discrete drivers in the same enclosure volume is not commonly found in these speaker design programs. There are work arounds that have been used for years, for example modeling the two drivers as an equivalent single driver http://www.quarter-wave.com/General/Two_Drivers.pdf. Taking the next step and explicitly modeling the interactions and responses of two discrete drivers sharing the same enclosure volume is a step up in complexity.

The intent of this presentation is to show the method used in some of my latest MathCad models for multiple drivers loading a common acoustic volume, present a transmission line enclosure example, and discuss the next steps for extending this method to more complicated loudspeaker systems.

Definition of One and Two Woofer TL Speakers



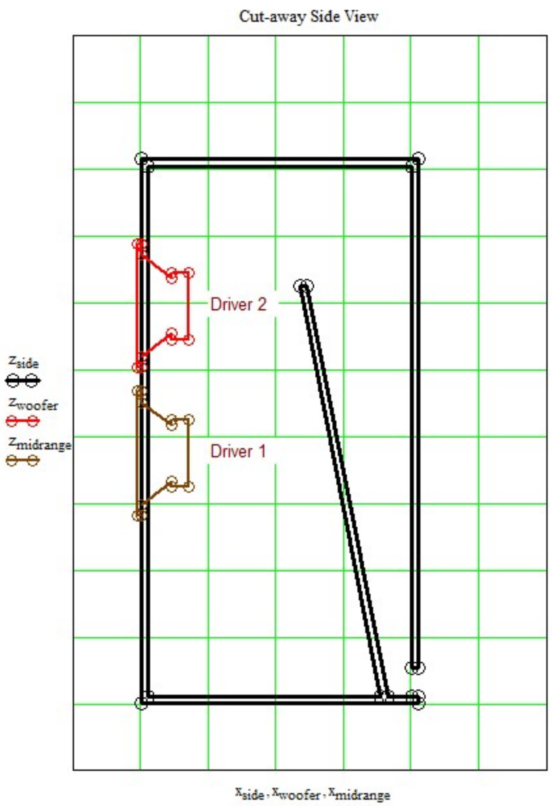


- On the left is a single 8” woofer in a 10:1 tapered transmission line. On the right is a pair of the same 8” woofers in an equivalent 10:1 tapered transmission line.
- For the two-driver enclosure, the tuning frequency (length and profile) is the same while the internal volume has been doubled. Increasing volume was accomplished by adding depth leaving the front baffle dimensions constant (width and height).
- The two TL lengths are approximately equal.
- The single woofer Is offset 30% along the length while the average position of the dual woofers is also offset 30% along the length.
- No damping material is used in the simulations to allow the peaks and dips to be easily observed in the acoustic impedance and SPL response plots.
- Both TL speaker systems use a 1” tweeter and an active crossover to complete the design.

Algorithm Derivation

The folded geometry of the 10:1 tapered TL is laid out in a straight line and shown above, The positions of Drivers 1 and 2 are shown along with the closed end and open-end denoted as points 0 and 3 respectively. From left to right the important points 0, 1, 2, and 3 will be used in the following derivation and discussion.

Modeling a single offset driver in a classic TL geometry has been derived elsewhere on this site. What is new for dual offset drivers is that they both interact with the air volume <u>and</u> each one can “feel” the influence of the other. There are two offsets that interact differently with the higher frequency standing waves so the driver motions will not always be identical.



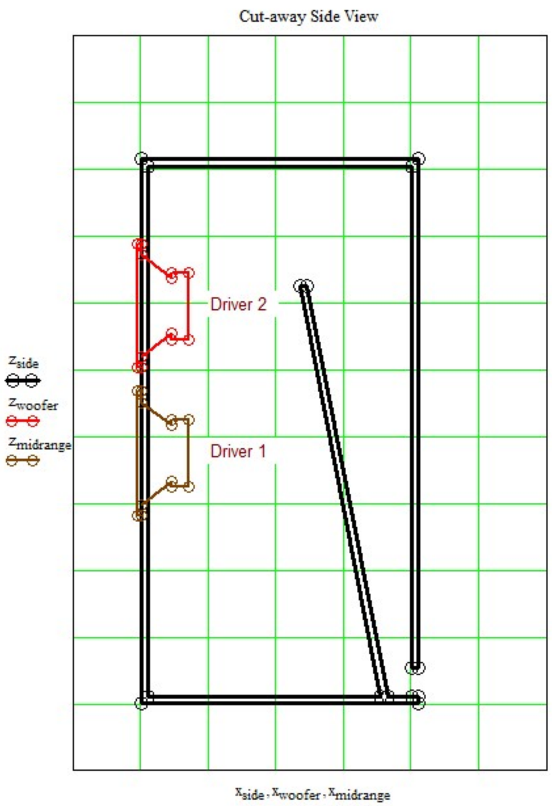
To construct and solve an equivalent electrical or acoustic circuit model for two drivers in a shared enclosure, the acoustic impedance acting on the back of each driver's cone is needed as a function of frequency. Acoustic impedance is defined as the ratio of pressure to volume velocity (area x velocity).

$$Z_{acoustic} = p / U = p / (\text{area x u})$$

The acoustic impedance of a sealed enclosure, using the Thiele Small models, is a simple capacitor and for a ported enclosure a simple capacitor and inductor. These lumped parameter models treat the pressure and volume velocity in the volume(s) as uniform.

For a transmission line or horn system, the acoustic impedance is more complicated. The 1D wave equation must be solved with appropriate boundary conditions at the open-end. The acoustic impedance becomes a complex function of frequency; it exhibits peaks and dips in magnitude and large swings in phase as it passes through the different standing wave resonant frequencies. The pressure and volume velocity vary along the length and depending on the geometry parallel paths may be needed to model the entire enclosure's air volume.

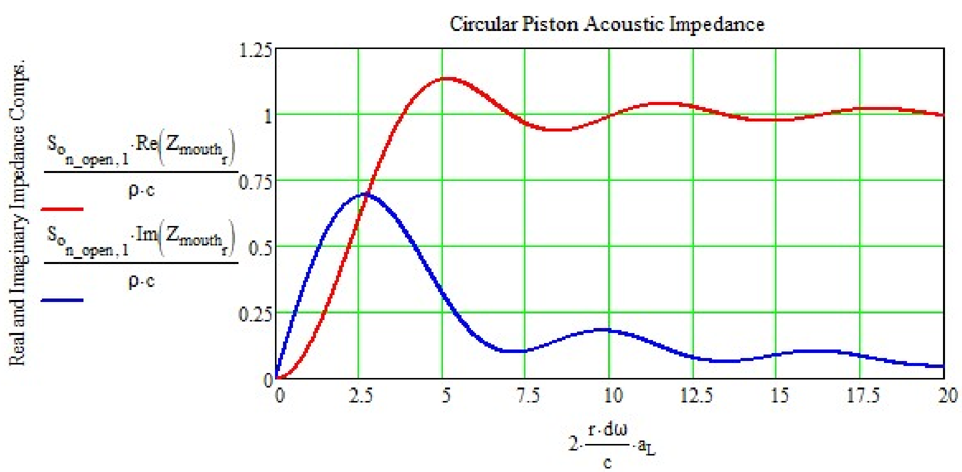
The acoustic impedance for the sealed, ported, transmission line, and horn enclosures with a single woofer or mid-woofer have all been derived, documented, and programmed in readily available simulation software.



To calculate the required acoustic impedances for two drivers in a common acoustic volume the properties of the drivers are not needed. Based on the geometry (length and cross-sectional area distribution) the goal is to characterize the pressure and volume velocity distributions along the length of the TL.

A simple mechanical analogy to this process is calculating the stiffness of a linear spring, fix one end and apply a small unit force to the other end while measuring the associated deflection. The ratio of force to deflection is the spring's linear stiffness. It does not matter how much force was applied; the stiffness will be a constant and a physical property of the spring. With the stiffness determined the response to any applied force can be calculated.

The starting point for calculating the acoustic impedances of the dual driver TL is to assume an acoustic impedance at the open-end, the lower right corner in the sketch. Usually, the acoustic impedance of a circular piston vibrating in an infinite baffle is assumed, different shapes and baffle configurations can also be used but the results will be very similar. Then if you apply a unit volume velocity the associated pressure is known, you can now work your way up the TL from the open-end towards the drivers. Transfer matrices between points 0, 1, 2, and 3 are used to calculate the required pressures and volume velocities at each point based on the assumed acoustic impedance at the open-end. These pressures and volume velocities are used to calculate enclosure's acoustic impedance matrix.



The acoustic impedance of a vibrating circular piston in an infinite baffle is show on the left. This plot should look familiar and can be found in most Acoustics textbooks.

Assume :

$U_{mouth} = U_3 = 1$

$p_{mouth} = p_3 = Z_{mouth} \times U_3$ so **p_3 and U_3 are now known**.

Define the Transfer Matrices for each section as follows :

$[U_i \;\; p_i]^T = [Transfer_{i<--j}] \times [U_j \;\; p_j]^T$ where i = 0, 1, 2 and j = 1, 2, 3.

The 2 x 2 matrix $[Transfer_{i<--j}]$ is moving from point j to point i

so in the opposite direction from point i to point j

$[U_j \;\; p_j]^T = [Transfer_{i<--j}]^{-1} \times [U_i \;\; p_i]^T$.

Transfer matrices are now available for whichever direction you are moving along the TL's length. Transfer matrices have been derived in previous TL theory documentation. There are other definitions for transfer matrices depending on what assumptions are made for the geometry of each section.

The three transfer matrices needed are shown below. These are calculated by a summation of the transfer matrices for each of the sections defined between the circular points, this process is also shown in the TL theory documentation on this site.

$[Transfer_{2<--3}]$

$[Transfer_{1<--2}]$

$[Transfer_{0<--1}]$

These transfer matrices determine the upstream point's pressure and volume velocity resulting from the downstream point's pressure and volume velocity.

One other relationship is needed to complete the algorithm relating the volume velocities transitioning past a moving driver.

This last relationship defines what happens as you pass each driver location while moving from point 3 to point 0.

For example, at point 2 :

p_2 is calculate and acts everywhere across point 2, on both sides of the driver.

The volume velocity splits if driver 2's cone is moving.

$U_2'' = U_2 - U_2'$ where

U_2 = volume velocity arriving from 3
U_2' = volume velocity of driver 2's cone
U_2'' = volume velocity leaving towards 1

U_2' is the cone volume velocity needed to produce U_3 and p_3. Note, the volume velocity towards the closed end, points 1 and 0, is not necessarily equal to the volume velocity arriving from point 3.

To finish the derivation, superposition will be used. Each driver's volume velocity and pressure will be treated independently and then combined.

Driver 1 : Driver 2's motion is zero.

$[U_2 \ \ p_2]^T = [Transfer_{2<--3}] \times [\mathbf{U_3 \ \ p_3}]^T$

Since $U_2' = 0$, $U_2'' = U_2$

$[U_1 \ \ p_1]^T = [Transfer_{1<--2}] \times [U_2'' \ \ p_2]^T$

Calculating the impedance towards the closed end as seen from point 1.

$[U \ \ p]^T = [Transfer_{0<--1}]^{-1} \times [0 \ \ 1]^T$

$Z_{closed} = p / U$

$U_1'' = p_1 / Z_{closed}$

$U_1' = U_1 - U_1''$ volume velocity of driver 1

Driver 1 : continued

Using p_1, U_1', and p_2 the first row's terms in the enclosure's acoustic impedance matrix can be calculated.

$Z_{11} = p_1 / U_1'$

$Z_{12} = p_2 / U_1'$

The last expression needed is the velocity ratio between driver 1 and the mouth.

$\varepsilon_1 = (S_{d\_1} / S_{mouth}) \times (1 / U_1')$

Driver 2 : Driver 1's motion is zero.

$[U_2 \;\; p_2]^T = [\text{Transfer}_{2<\text{--}3}] \times [\mathbf{U_3 \;\; p_3}]^T$

Since $U_1' = 0$, $U_1'' = U_1$

Calculating the impedance towards the closed end as seen from point 2.

$[T_{2<\text{--}0}] = [\text{Transfer}_{1<\text{--}2}]^{-1} \times [\text{Transfer}_{0<\text{--}1}]^{-1}$

$[U \;\; p]^T = [T_{2<\text{--}0}] \times [0 \;\; 1]^T$

$Z_{closed} = p / U$

$U_2'' = p_2 / Z_{closed}$

$U_2' = U_2 - U_2''$ volume velocity of driver 2

$[U_1 \;\; p_1]^T = [\text{Transfer}_{1<\text{--}2}] \times [U_2'' \;\; p_2]^T$

U_1 should be zero if calculated correctly,

Driver 2 : continued

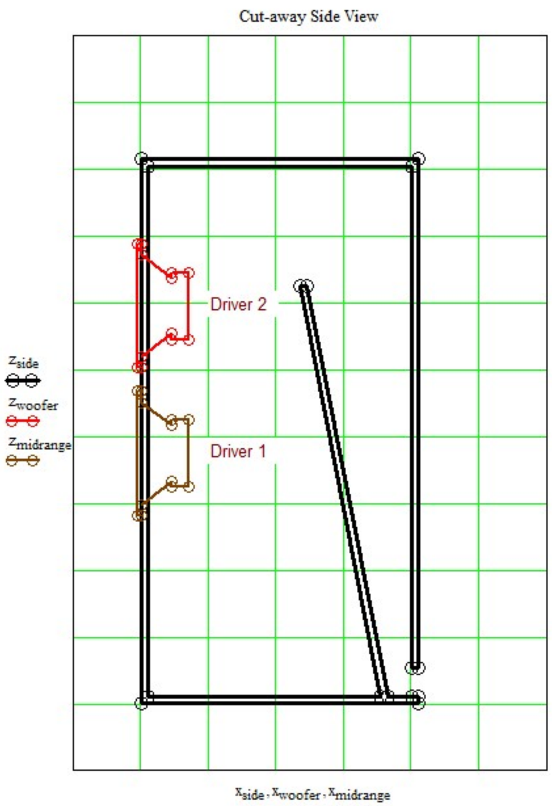
Using p_2, U_2', and p_1 the second row's terms in the enclosure's acoustic impedance matrix can be calculated.

$Z_{21} = p_1 / U_2'$

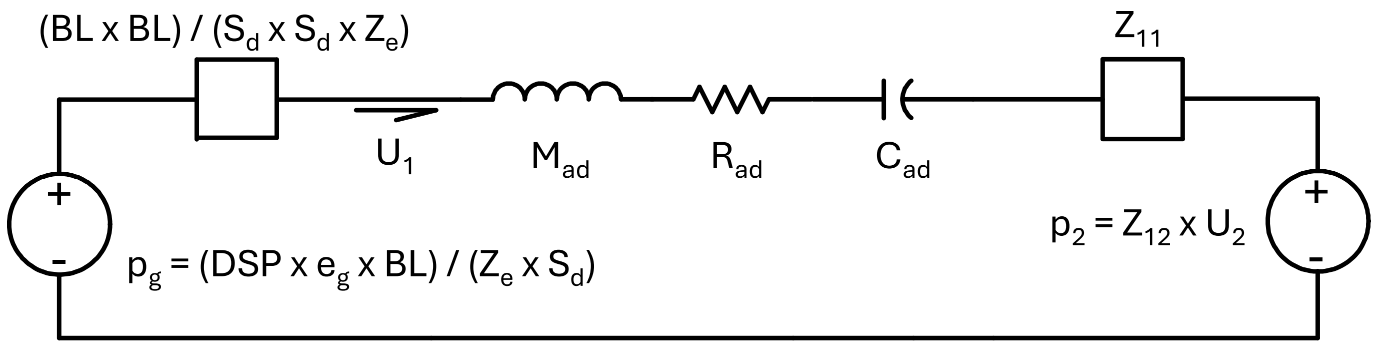
$Z_{22} = p_2 / U_2'$

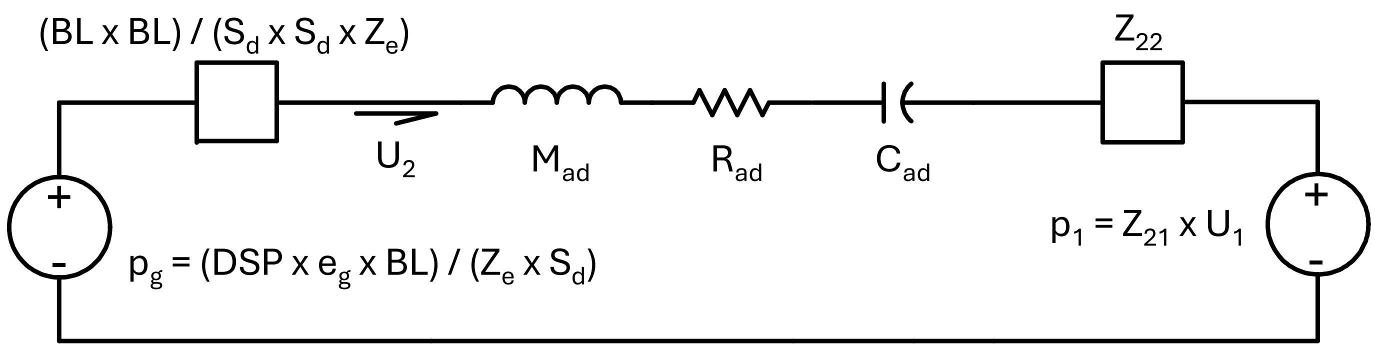
The last expression needed is the velocity ratio between driver 2 and the mouth.

$\varepsilon_2 = (S_{d\_2} / S_{mouth}) \times (1 / U_2')$



To calculate the actual volume velocities for Driver 1, Driver 2, and the open-end of the TL a pair of coupled equations were programmed in MathCad. In the math the "mid" driver is Driver 1 (closer to the closed end of the TL) while the "low" driver is Driver 2 (further offset along the length of the TL). The open-end has the "L" subscript. The worksheet was originally set up to analyze a three-way TL with low, mid, and high frequency drivers. The mid and low frequency drivers can be defined as the same to simulate a dual mid-woofer and tweeter configuration.





Equivalent Acoustic Circuits :

Equivalent acoustic circuits for two identical drivers in a common TL enclosure are shown on the left.

An additional voltage source, p_1 or p_2, is inserted on the right side of each circuit to represent the pressure on the back of one driver cone resulting from the other driver cone's motion.

Removing the second driver reduces the circuit to the equivalent acoustic circuit for a single driver in a TL.

There will now be two equations with two unknowns, U_1 and U_2, that need to be solved as a function of frequency.

Equivalent Acoustic Circuits : MathCad Programming Details

Acoustic Impedance Matrix Elements

$$Z_{one\_one_r} := \left(\frac{Bl_{mid}^{2}}{S_{d\_mid}^{2} \cdot Z_{e\_mid_r}} + \frac{1}{j \cdot r \cdot d\omega \cdot C_{ad\_mid}} + R_{ad\_mid} + j \cdot r \cdot d\omega \cdot M_{ad\_mid} + Z_{mid\_mid_r} \right)$$

$$Z_{two\_two_r} := \left(\frac{Bl_{low}^{2}}{S_{d\_low}^{2} \cdot Z_{e\_low_r}} + \frac{1}{j \cdot r \cdot d\omega \cdot C_{ad\_low}} + R_{ad\_low} + j \cdot r \cdot d\omega \cdot M_{ad\_low} + Z_{low\_low_r} \right)$$

$$Z_{one\_two_r} := Z_{mid\_low_r} \qquad Z_{two\_one_r} := Z_{low\_mid_r}$$

Solving for Cone Volume Velocities

$$Z_r := \begin{pmatrix} Z_{one\_one_r} \cdot \frac{m^3}{Pa \cdot sec} & Z_{one\_two_r} \cdot \frac{m^3}{Pa \cdot sec} \\ Z_{two\_one_r} \cdot \frac{m^3}{Pa \cdot sec} & Z_{two\_two_r} \cdot \frac{m^3}{Pa \cdot sec} \end{pmatrix}$$

$$\begin{pmatrix} U_{mid_r} \\ U_{low_r} \end{pmatrix} := \left(Z_r\right)^{-1} \cdot \begin{pmatrix} MP_{boost} \cdot MP_{phase} \cdot M\_P_r \cdot p_{g\_mid_r} \\ SHP_{SHP_{order},r} \cdot LP_{boost} \cdot LP_{phase} \cdot L\_P_r \cdot p_{g\_low_r} \end{pmatrix} \cdot \left(\frac{Pa \cdot sec}{m^3} \right)^{-1}$$

$$U_{L\_mid_r} := -\varepsilon_{mid_r} \cdot \frac{S_{o_{n\_open,1}}}{S_{d\_mid}} \cdot U_{mid_r}$$

$$U_{L\_low_r} := -\varepsilon_{low_r} \cdot \frac{S_{o_{n\_open,1}}}{S_{d\_low}} \cdot U_{low_r}$$

$$U_{L_r} := U_{L\_mid_r} + U_{L\_low_r}$$

The equations in matrix notation

$$[p_1 \;\; p_2]^T = [Z] \times [U_1 \;\; U_2]^T$$

$[p_1 \;\; p_2]^T$ = applied p_g from amplifier through DSP crossover
$[Z]$ = acoustic impedance matrix for the air volume in the TL
$[U_1 \;\; U_2]^T$ = unknown volume velocities of drivers 1 (mid) and 2 (low)

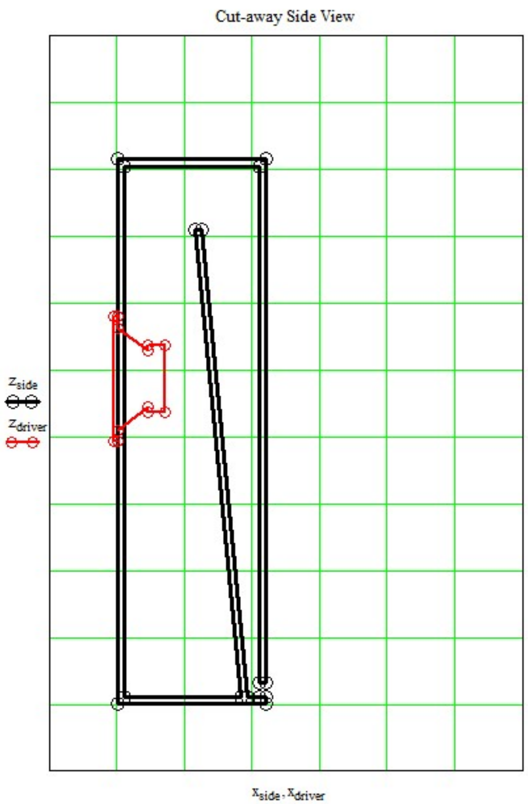
Solving

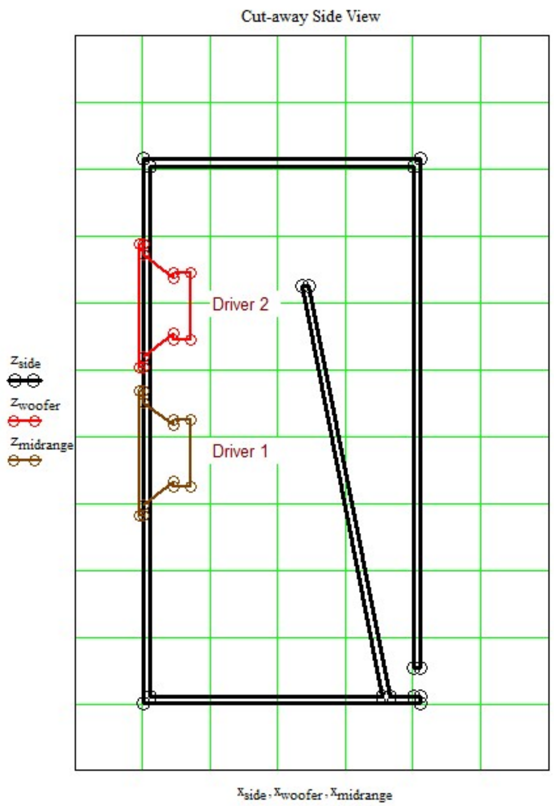
$$[U_1 \;\; U_2]^T = [Z]^{-1} \times [p_1 \;\; p_2]^T$$

Knowing U_1 and U_2 the volume velocity at the open-end U_L can be calculated for each driver and then added using the superposition principle.

Passive crossovers can also be used in this model, but it is a little more complex and details will not be provided at this time.

One and Two Woofer TL Speakers - Example





Repeating the Information from Slide 3

- On the left is a single 8” woofer in a 10:1 tapered transmission line. On the right is a pair of the same 8” woofers in an equivalent 10:1 tapered transmission line.
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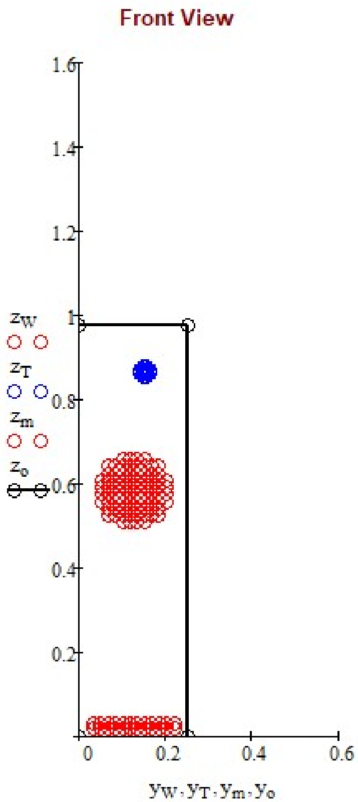
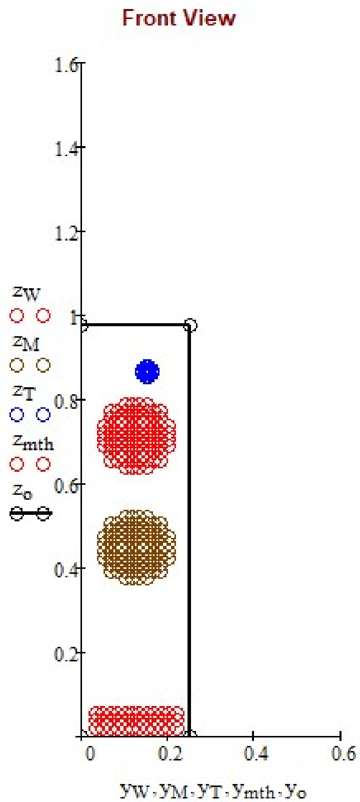
- No damping material is used in the simulations to allow the peaks and dips to be easily observed in the acoustic impedance and SPL response plots.
- Both TL speaker systems use a 1” tweeter and an active crossover to complete the design.

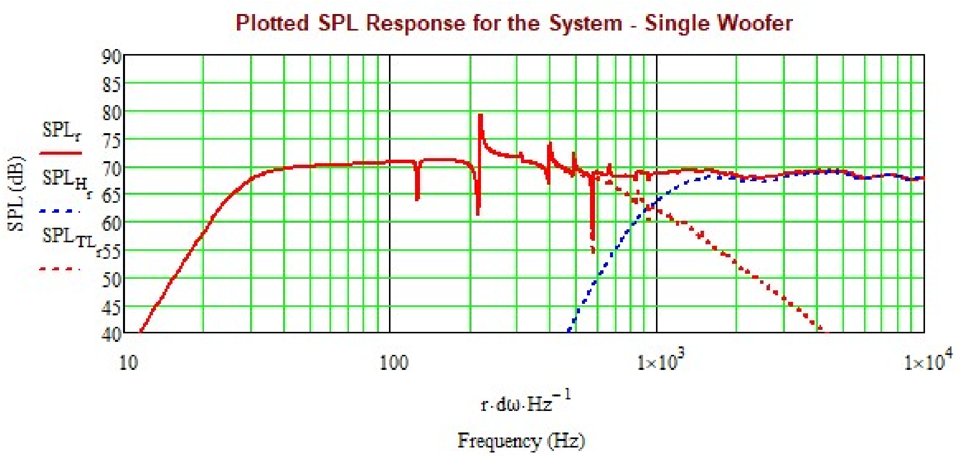
Single Woofer

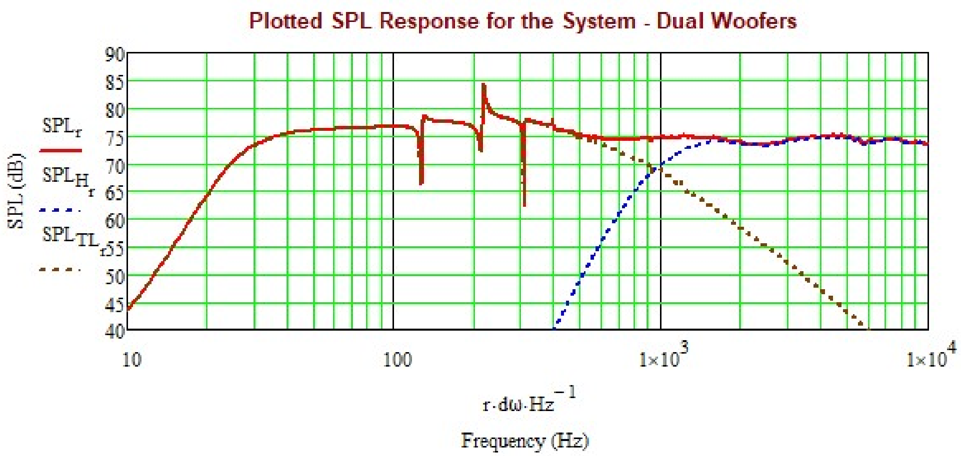


Dual Woofers

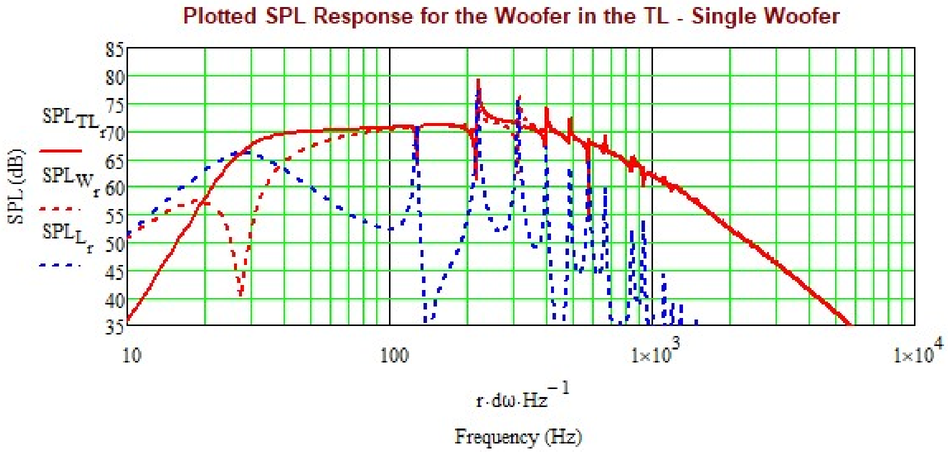


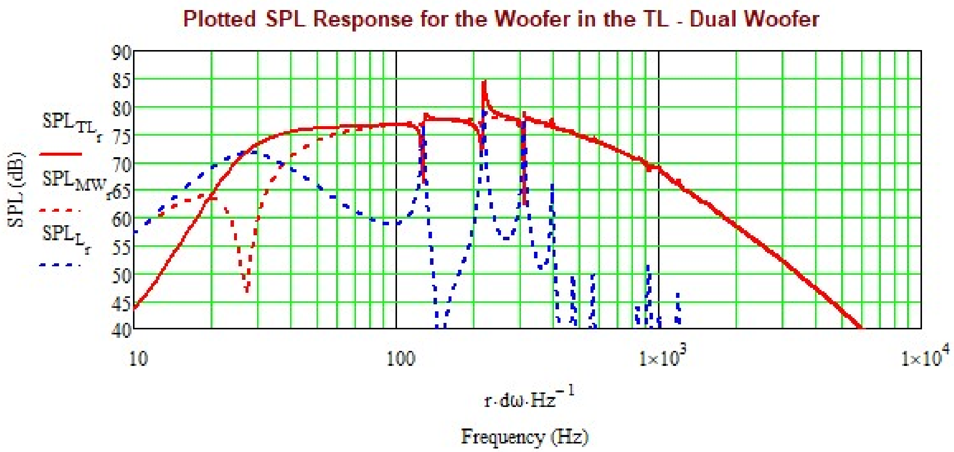
- Front view of the single and dual woofer TL speaker systems.
- Baffle dimensions are the same.
- Tweeter position is the same.
- Active crossovers are the same except for the amount of padding down applied to the tweeter.
- The average driver offset position for the dual woofer TL system is the same as the driver offset for the single woofer TL system.
- The open-end is located on the bottom of the rear baffle.
- While the fronts of the two TL speaker systems are very similar, the open-end size and depth location is larger in the dual woofer TL speaker. This had a minor impact.
- Comparing the SPL responses for two speakers should be a reasonable comparison and a valid indicator of the performance similarities and differences between them.



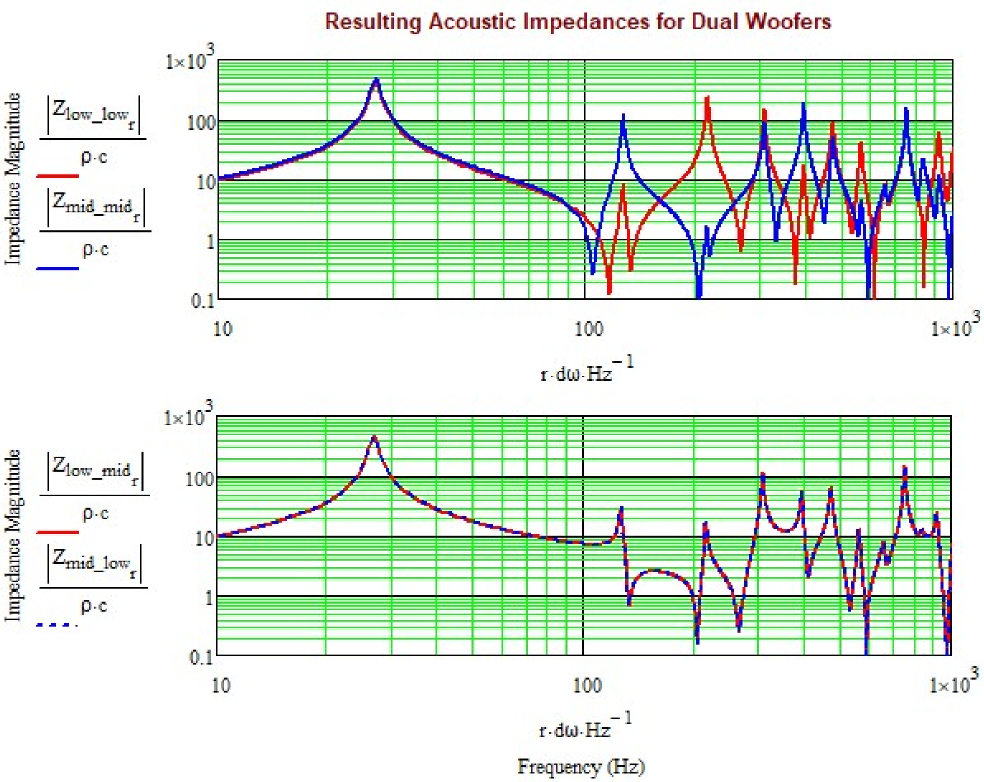


- The SPL responses at 3 m are plotted for the single woofer (top plot) and dual woofer (bottom plot) TL speaker systems.
- The results are very similar at low frequencies; the dual woofer SPL is 6 dB higher as expected for two drivers wired in parallel.
- Tweeter SPL response is the same but 6 dB higher, amount of padding was reduced in the active crossover, for the dual woofer TL speaker system.
- While the SPL responses are similar in shape below 300 Hz, above 300 Hz the dual woofer design has significantly fewer peaks and nulls.





- The contributions to the TL's SPL response from the driver(s) (red dashed curve) and the open-end (blue dashed curve) are split and plotted.
- The scale on the single driver SPL plot is adjusted so that the curves in the two results occupy the same positions in both plots.
- The SPL responses are similar in shape below 300 Hz, above 300 Hz the dual woofer design has significantly less open-end SPL output.
- Using two smaller drivers in a TL might be another tool for taming the higher quarter-wave modes found in classic TL speaker systems. It might be another method used to reduce the amount of damping material required preserving more of the low-end output.



- For completeness, the acoustic impedances seen by the back of the dual woofer cones are plotted.
- The red curves are the “low” driver or Driver 2 and the blue curves are the “mid” driver or Driver 1 in the cross-section view shown on slide 17.
- The top plot presents the direct acoustic impedances, the back pressure on each driver’s cone due to its own motion.
- The lower plot presents the coupling acoustic impedances, the back pressure on one driver’s cone due to the motion of the other driver’s cone. As expected for a linear system, these are the same curves.

Take Aways and Next Steps

- The intent of this presentation was to lay out the method for modeling two drivers sharing the same acoustic volume, slides 4 through 16. The method has been extended to three driver.
- Slides 17 through 21 presented an example problem comparing results for equivalent single and dual driver TLs. Some of the results for the dual driver version were expected (+6 dB of SPL output) and a few were a nice surprise (better control of peaks and dips produced by higher harmonics).
- One path forward is to further study and optimize TL designs using two woofers or mid-woofer drivers, but that is probably going to wait for another time.

- The MathCad model in the worksheet described in this presentation was a simplification of one of the worksheets I have put together over the past year to model Multi Entry Horn (MEH) designs.
- I need to do some more clean-up and checking but believe in the future to have worksheets that will simulate two- and three-way MEHs with interacting drivers and either active or passive crossovers. Unless I get distracted again (which is very possible), this body of work will probably result in a separate page on my site that will continue to evolve as my models and understanding improves for this fascinating type of horn speaker.
- Maybe theory will even go the next big step and become a physical speaker build. That will represent a huge challenge for me and my limited hands-on skill set, so don't hold your breath.